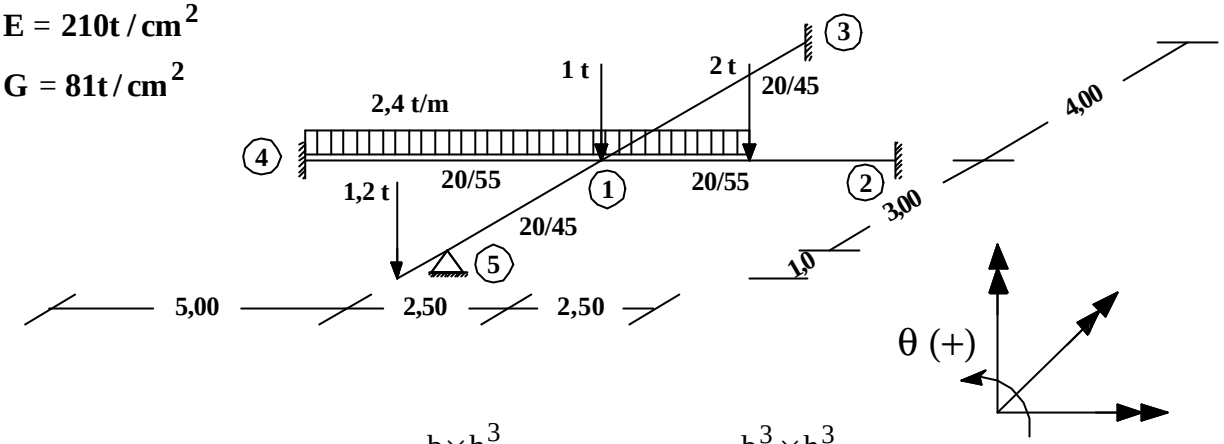
EMPARRILLADO DE VIGA

$E = 210 t/cm^2$

$G = 81 t/cm^2$

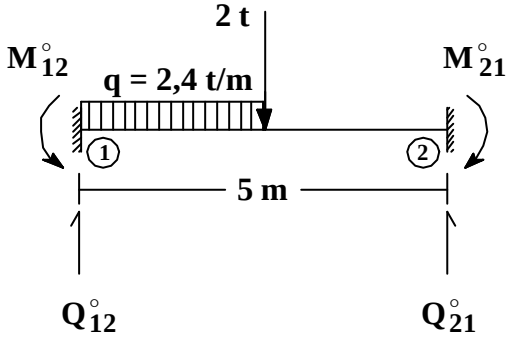


Cálculos Auxiliares

$$I_f = \frac{b \times h^3}{12} \qquad I_f = \frac{b^3 \times h^3}{3{,}6(b^2 + h^2)}$$

Barra	L (cm)	θ	Sen θ	Cos θ	I_f (cm^4)	I_t (cm^4)	$G.I_t / L$	$E.I_f / L$	$E.I_f / L^2$	$E.I_f / L^3$
1-2	500	0°	0	1	277292	107948	1748,6	116462,6	232,9	0,46
1-3	400	90°	1	0	151875	83505	16909,8	79734,4	199,3	0,50
1-4	500	180°	0	- 1	277292	107948	17487,6	116462,6	232,9	0,46
1-5	300	270°	- 1	0	151875	83505	22546,3	106312,5	354,4	1,18

Reacciones de Empotramiento Perfecto (A°_{ij})

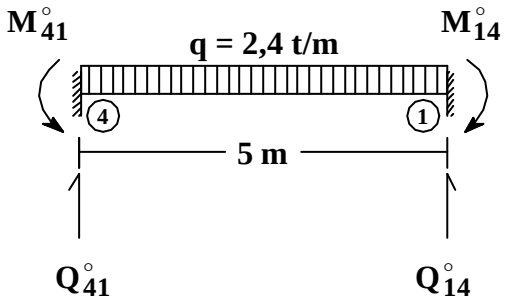


$$M^\circ_{12} = -\frac{P \times L}{8} - \frac{11}{192} \times q \times L^2 = -4{,}68 tm$$

$$M^\circ_{21} = \frac{P \times L}{8} + \frac{5}{192} \times q \times L^2 = 2{,}81 tm$$

$$Q^\circ_{12} = \frac{P}{2} + \frac{13}{32} \times q \times L = 5{,}875 t$$

$$Q^\circ_{12} = \frac{P}{2} + \frac{3}{32} \times q \times L = 2{,}12 t$$

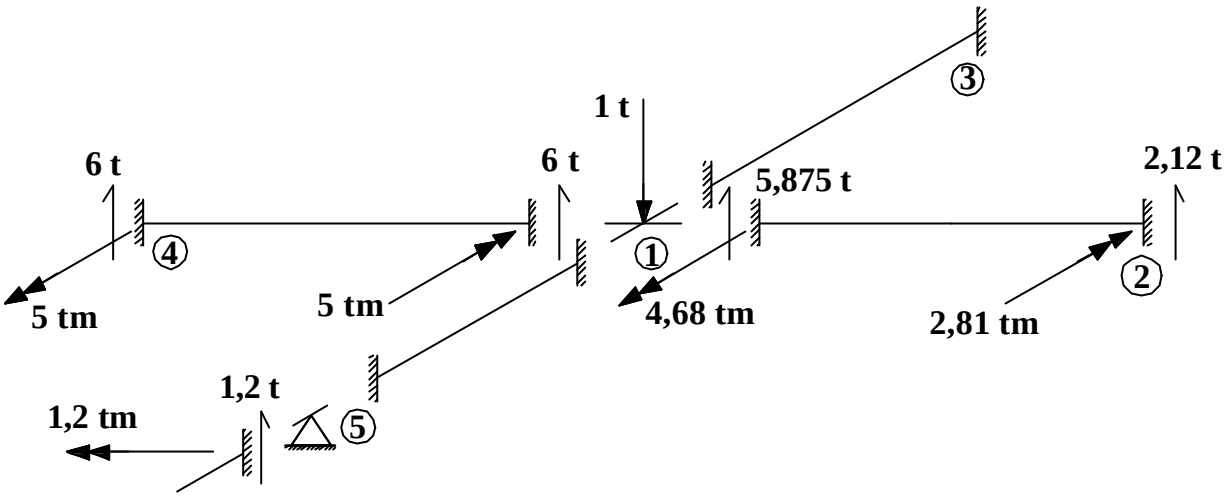


$$M^\circ_{14} = \frac{q \times L^2}{12} = 5 tm$$

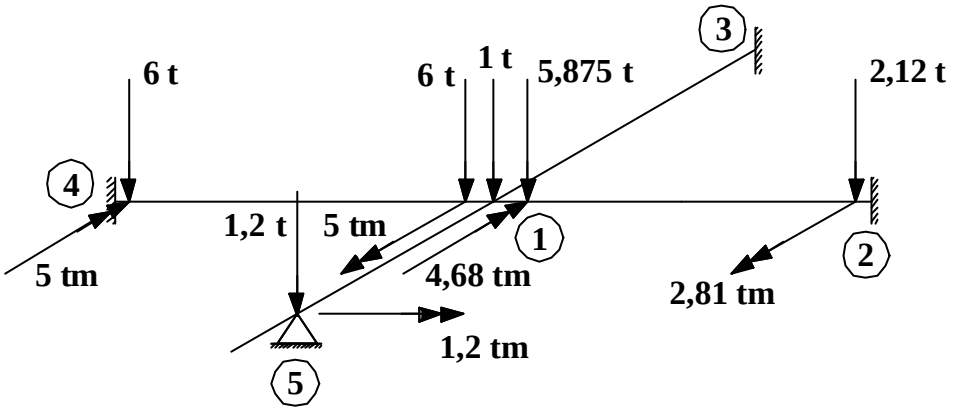
$$M^\circ_{41} = -5 tm$$

$$Q^\circ_{14} = Q^\circ_{41} = \frac{q \times L^2}{12} = 6 t$$

Estructura Bloqueada



Estructura con Cargas Nodales Equivalentes



Matriz de Rigidez de Barras

- Barra 1-2: (nudo 2 empotrado)

$$\begin{vmatrix} \underset{\sim}{S_{11}} & \underset{\sim}{S_{12}} \\ \underset{\sim}{S_{21}} & \underset{\sim}{S_{22}} \end{vmatrix} = \begin{vmatrix} 17487{,}6 & 0 & 0 \\ 0 & 465850 & -1397{,}6 \\ 0 & -1397{,}6 & 5{,}59 \\ 17487{,}6 & 0 & 0 \\ 0 & 232925 & -1397{,}6 \\ 0 & 1397{,}6 & -5{,}59 \end{vmatrix}$$

- Barra 1-3: (nudo 3 empotrado)

$$\begin{vmatrix} \underset{\sim}{S_{11}} & \underset{\sim}{S_{13}} \\ \underset{\sim}{S_{31}} & \underset{\sim}{S_{33}} \end{vmatrix} = \begin{vmatrix} 318938 & 0 & 1196 \\ 0 & 16909,8 & 0 \\ 1196 & 0 & 5,98 \\ 159469 & 0 & 1196 \\ 0 & -16909,8 & 0 \\ -1196 & 0 & -5,98 \end{vmatrix}$$

- Barra 1-4: (nudo 4 empotrado)

$$\begin{vmatrix} \underset{\sim}{S_{11}} & \underset{\sim}{S_{14}} \\ \underset{\sim}{S_{41}} & \underset{\sim}{S_{44}} \end{vmatrix} = \begin{vmatrix} 17487,6 & 0 & 0 \\ 0 & 465850 & 1397,6 \\ 0 & 1397,6 & 5,59 \\ -17487,6 & 0 & 0 \\ 0 & 232925 & 1397,6 \\ 0 & -1397,6 & -5,59 \end{vmatrix}$$

- Barra 1-3: (nudo 3 empotrado)

$$\begin{vmatrix} \underset{\sim}{S_{11}} & \underset{\sim}{S_{13}} \\ \underset{\sim}{S_{31}} & \underset{\sim}{S_{33}} \end{vmatrix} = \begin{vmatrix} 425250 & 0 & -2126,3 & 212625 & 0 & 2126,3 \\ 0 & 22546,4 & 0 & 0 & -22546,4 & 0 \\ -2126,3 & 0 & 14,18 & -2126,3 & 0 & -14,18 \\ 212625 & 0 & -2126,3 & 425250 & 0 & 2126,3 \\ 0 & -22546,4 & 0 & 0 & 22546,4 & 0 \\ 2126,3 & 0 & -14,18 & 2126,3 & 0 & 14,18 \end{vmatrix}$$

Condiciones de Vínculos

$$\underset{\sim}{D_1} = \begin{bmatrix} \theta_{X1} \neq 0 \\ \theta_{Y1} \neq 0 \\ \Delta_{Z1} \neq 0 \end{bmatrix}; \quad \underset{\sim}{D_2} = \begin{bmatrix} \theta_{X2} = 0 \\ \theta_{Y2} = 0 \\ \Delta_{Z2} = 0 \end{bmatrix}; \quad \underset{\sim}{D_3} = \begin{bmatrix} \theta_{X3} = 0 \\ \theta_{Y3} = 0 \\ \Delta_{Z3} = 0 \end{bmatrix}; \quad \underset{\sim}{D_4} = \begin{bmatrix} \theta_{X4} = 0 \\ \theta_{Y4} = 0 \\ \Delta_{Z4} = 0 \end{bmatrix}$$

$$\underset{\sim}{D_5} = \begin{bmatrix} \theta_{X5} \neq 0 \\ \theta_{Y5} \neq 0 \\ \Delta_{Z5} = 0 \end{bmatrix}$$

Sistema Ensamblado

$$\begin{vmatrix} \underset{\sim}{S_{11}} & \underset{\sim}{S_{12}} & \underset{\sim}{S_{13}} & \underset{\sim}{S_{14}} & \underset{\sim}{S_{15}} \\ \underset{\sim}{S_{21}} & \underset{\sim}{S_{22}} & \underset{\sim}{S_{23}} & \underset{\sim}{S_{24}} & \underset{\sim}{S_{25}} \\ \underset{\sim}{S_{31}} & \underset{\sim}{S_{32}} & \underset{\sim}{S_{33}} & \underset{\sim}{S_{34}} & \underset{\sim}{S_{35}} \\ \underset{\sim}{S_{41}} & \underset{\sim}{S_{42}} & \underset{\sim}{S_{43}} & \underset{\sim}{S_{44}} & \underset{\sim}{S_{45}} \\ \underset{\sim}{S_{51}} & \underset{\sim}{S_{52}} & \underset{\sim}{S_{53}} & \underset{\sim}{S_{54}} & \underset{\sim}{S_{55}} \end{vmatrix} = \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \Delta_{Z2} \\ \theta_{X3} \\ \theta_{Y3} \\ \Delta_{Z3} \\ \theta_{X4} \\ \theta_{Y4} \\ \Delta_{Z4} \\ \theta_{X5} \\ \theta_{Y5} \\ \Delta_{Z5} \end{vmatrix} \times \begin{vmatrix} \underset{\sim}{A_1} \\ \underset{\sim}{A_2} \\ \underset{\sim}{A_3} \\ \underset{\sim}{A_4} \\ \underset{\sim}{A_5} \end{vmatrix}$$

$$\underset{\sim}{S_{11}} = \underset{\sim}{S_{11}^{(1-2)}} + \underset{\sim}{S_{11}^{(1-3)}} + S_{11}^{(1-4)} + S_{11}^{(1-5)}$$

Sistema Resultante (con condiciones de vínculo):

$$\begin{vmatrix} 779163 & 0 & - 930,2 & 212625 & 0 \\ 0 & 971156 & 0 & 0 & - 22546,4 \\ - 930,2 & 0 & 31,3 & - 2126,3 & 0 \\ 212625 & 0 & - 2126,3 & 425250 & 0 \\ 0 & - 22546,4 & 0 & 0 & 22546,4 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \\ \theta_{X5} \\ \theta_{Y5} \end{vmatrix} = \begin{vmatrix} 0 \\ - 32\ tcm \\ - 12,875\ t \\ 120\ tcm \\ 0 \end{vmatrix}$$

Resolviendo:

$\theta_{X1} = 2,79576 \times 10^{-5}$ rad.
$\theta_{Y1} = - 3,37336 \times 10^{-5}$ rad.
$\Delta_{Z1} = - 0,593054$ cm
$\theta_{X5} = - 2,69706 \times 10^{-5}$ rad.
$\theta_{Y5} = - 3,37336 \times 10^{-5}$ rad.

Calculo de los Esfuerzos Extremos de Barra

$$F_{ij} = S_{ii} \times D_i + S_{ij} \times D_j + A_{ij}^{\circ} \qquad \text{(Acciones nudo sobre barra)}$$

- Barra 1-2:

$$\underset{\sim}{F_{12}} = \underset{\sim}{S_{11}} \times \underset{\sim}{D_1} + \underset{\sim}{S_{12}} \times \overset{\approx 0}{\underset{\sim}{D_2}} + \underset{\sim}{A_{12}^{\circ}}$$

$$\begin{vmatrix} 1748,6 & 0 & 0 \\ 0 & 465850 & -1397,6 \\ 0 & -1397,6 & 5,59 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 0 \\ -468 \\ 5,875 \end{vmatrix} = \begin{vmatrix} M_{X12} = 0,5 \text{ tcm} \\ M_{Y12} = 345,11 \text{ tcm} \\ Q_{12} = 2,61 \text{ t} \end{vmatrix}$$

$$\underset{\sim}{F_{21}} = \underset{\sim}{S_{22}} \times \overset{\approx 0}{\underset{\sim}{D_2}} + \underset{\sim}{S_{21}} \times \underset{\sim}{D_1} + \underset{\sim}{A^{\circ}_{21}}$$

$$\begin{vmatrix} 1748,6 & 0 & 0 \\ 0 & 232925 & -1397,6 \\ 0 & 1397,6 & -5,59 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 0 \\ 281 \\ 2,12 \end{vmatrix} = \begin{vmatrix} M_{X21} = -0,5 \text{ tcm} \\ M_{Y21} = 1102 \text{ tcm} \\ Q_{21} = 5,39 \text{ t} \end{vmatrix}$$

- Barra 1-3:

$$\underset{\sim}{F_{13}} = \underset{\sim}{S_{11}} \times \underset{\sim}{D_1} + \underset{\sim}{S_{13}} \times \overset{\approx 0}{\underset{\sim}{D_3}} + \underset{\sim}{A^{\circ}_{13}}$$

$$\begin{vmatrix} 318938 & 0 & 1196 \\ 0 & 16909,8 & 0 \\ 1196 & 0 & 5,98 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} M_{X13} = -700,4 \text{ tcm} \\ M_{Y13} = -0,57 \text{ tcm} \\ Q_{12} = -3,51 \text{ t} \end{vmatrix}$$

$$\underset{\sim}{F_{31}} = \underset{\sim}{S_{33}} \times \overset{\approx 0}{\underset{\sim}{D_3}} + \underset{\sim}{S_{31}} \times \underset{\sim}{D_1} + \underset{\sim}{A^{\circ}_{31}}$$

$$\begin{vmatrix} 159469 & 0 & 1196 \\ 0 & -16909,8 & 0 \\ -1196 & 0 & -5,98 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} M_{X31} = -704,8 \text{ tcm} \\ M_{Y31} = 0,57 \text{ tcm} \\ Q_{31} = 3,51 \text{ t} \end{vmatrix}$$

- Barra 1-4:

$$\underset{\sim}{F_{14}} = \underset{\sim}{S_{11}} \times \underset{\sim}{D_1} + \underset{\sim}{S_{14}} \times \overset{\approx 0}{\underset{\sim}{D_4}} + \underset{\sim}{A^{\circ}_{14}}$$

$$\begin{vmatrix} 17487,6 & 0 & 0 \\ 0 & 465850 & 1397,6 \\ 0 & 1397,6 & 5,59 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 0 \\ 500 \\ 6 \end{vmatrix} = \begin{vmatrix} M_{X14} = 0,50 \text{ tcm} \\ M_{Y14} = -344,54 \text{ tcm} \\ Q_{14} = 2,63 \text{ t} \end{vmatrix}$$

$$\underset{\sim}{F_{41}} = \underset{\sim}{S_{44}} \times \overset{\approx 0}{\underset{\sim}{D_4}} + \underset{\sim}{S_{41}} \times \underset{\sim}{D_1} + \underset{\sim}{A^{\circ}_{41}}$$

$$\begin{vmatrix} -17487,6 & 0 & 0 \\ 0 & 232925 & 1397,6 \\ 0 & -1397,6 & -5,59 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 0 \\ -500 \\ 6 \end{vmatrix} = \begin{vmatrix} M_{X41} = -0,50 \text{ tcm} \\ M_{Y41} = -1336,7 \text{ tcm} \\ Q_{41} = 9,37 \text{ t} \end{vmatrix}$$

- Barra 1-5:

$$\underset{\sim}{F_{15}} = \underset{\sim}{S_{11}} \times \underset{\sim}{D_1} + \underset{\sim}{S_{15}} \times \underset{\sim}{D_5} + \underset{\sim}{A^{\circ}_{15}}$$

$$\begin{vmatrix} 425250 & 0 & -2126,3 \\ 0 & 22546,4 & 0 \\ -2126,3 & 0 & 14,18 \end{vmatrix} \times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 212625 & 0 & 2126,3 \\ 0 & -22546,4 & 0 \\ -2126,3 & 0 & -14,18 \end{vmatrix} \times$$

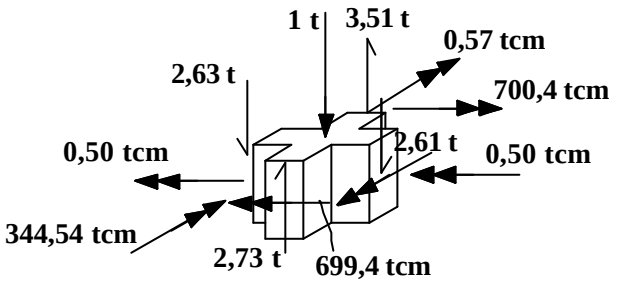
$$\times \begin{vmatrix} \theta_{X5} \\ \theta_{Y5} \\ \Delta_{Z5} \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} M_{X15} = 699,4 \text{ tcm} \\ M_{Y15} = 0 \\ Q_{15} = -2,73 \text{ t} \end{vmatrix}$$

$$\underset{\sim}{F_{51}} = \underset{\sim}{S_{55}} \times \underset{\sim}{D_5} + \underset{\sim}{S_{51}} \times \underset{\sim}{D_1} + \underset{\sim}{A^{\circ}_{51}}$$

$$\begin{vmatrix} 425250 & 0 & 2126,3 \\ 0 & 22546,4 & 0 \\ 2126,3 & 0 & 14,18 \end{vmatrix} \times \begin{vmatrix} \theta_{X5} \\ \theta_{Y5} \\ \Delta_{Z5} \end{vmatrix} + \begin{vmatrix} 212625 & 0 & -2126,3 \\ 0 & -22546,4 & 0 \\ 2126,3 & 0 & -14,18 \end{vmatrix} \times$$

$$\times \begin{vmatrix} \theta_{X1} \\ \theta_{Y1} \\ \Delta_{Z1} \end{vmatrix} + \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} M_{X51} = 120 \text{ tcm} \\ M_{Y51} = 0 \\ Q_{51} = 2,73 \text{ t} \end{vmatrix}$$

Verificación Equilibrio de Nudo (Acciones barra sobre nudo)



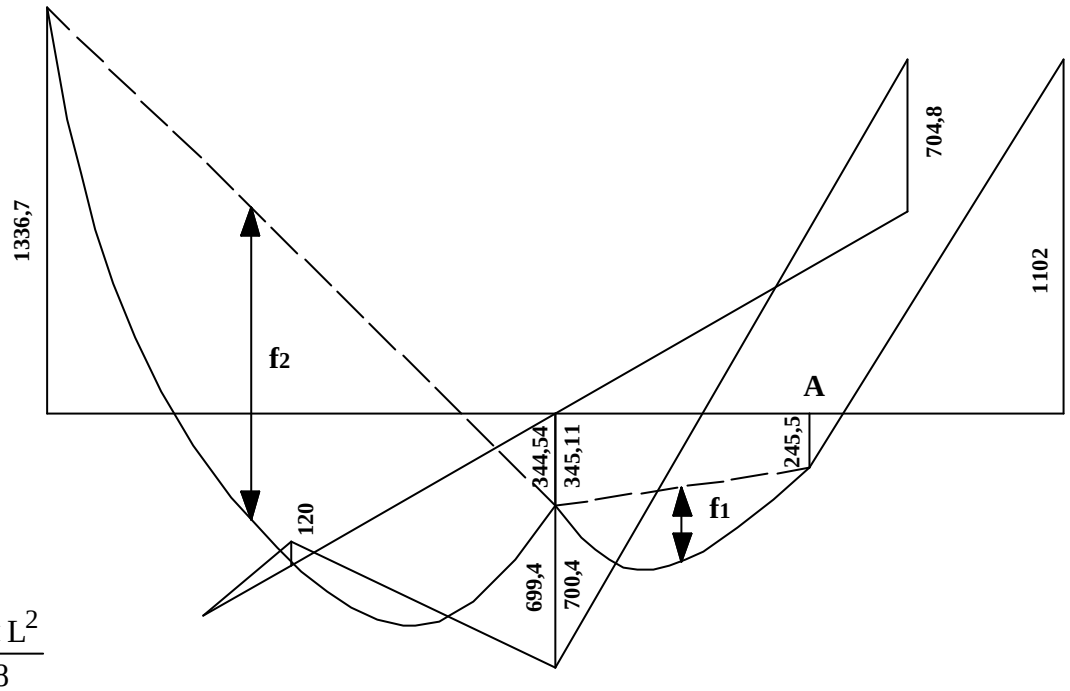
$\sum M_x = (-0,50 + 700,4 - 0,50 - 699,4)$ tcm = 0

$\sum M_y = (-345,11 + 0,57 + 344,54)$ tcm = 0

$\sum Q = (-2,61 + 3,51 - 2,63 + 2,73 - 1)$ t = 0

Diagramas Finales

- **Momento Flector: (tcm)**



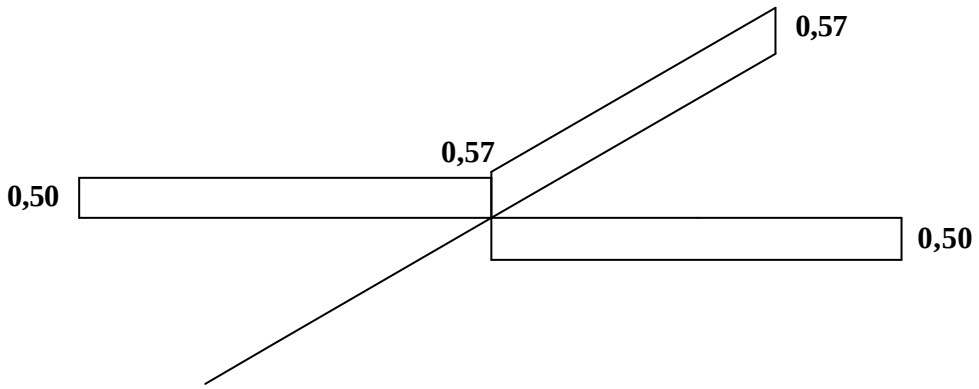
$$f = \frac{q \times L^2}{8}$$

$$f_1 = \frac{2{,}4t/m \times (2{,}5m)^2}{8} = 187{,}5tcm$$

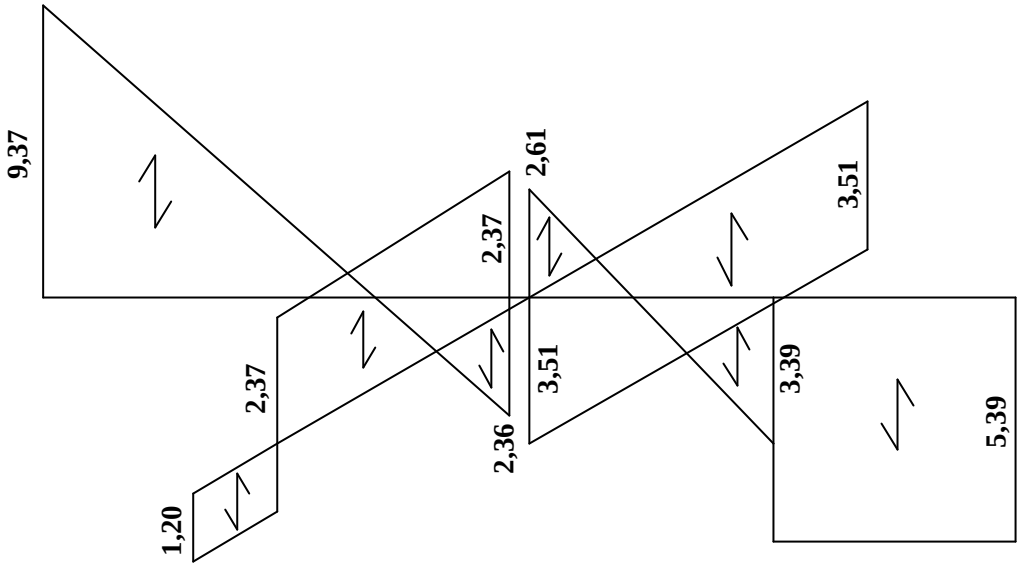
$$f_2 = \frac{2{,}4t/m \times (5m)^2}{8} = 750tcm$$

M_A = - 1102 tcm + 5,39 t × 250 cm = 245,5 tcm

- **Momento Torsor: (tcm)**



- **Esfuerzo de Corte: (t)**



- **Cuerpo Libre:**